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Detecting Strangeness -4 Dibaryon States

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Recent experiments at Jefferson Laboratory and potential new facilities at the Japan Proton Accelerator Research Complex (J-PARC) make it evident that the discovery of a $^1S_0$ di-cascade bound state of two Ξ particles is feasible. We state the simple arguments, based on $SU(3)$ flavor symmetry, for the existence of this bound state, review the previous predictions and comment on the experimental conditions necessary for detection.

The first measurement of exclusive doubly-strange cascade $\Xi^-(1321)$ hyperon production in the $\gamma p \to K^+K^+\Xi^-$ reaction at Jefferson Laboratory has recently been reported [1]. The high-quality photon beam was used to excite the narrow Ξ^- state that was observed as a sharp peak in the missing mass spectrum. This success opens the door to many avenues of research including double hypernuclear production.

The interest in understanding the properties of the cascade spectrum stem from QCD which, in its earliest incarnation, expressed the strong-interaction Hamiltonian as the sum of an $SU(3)$ invariant term and a medium strong interaction term (now known as the quark mass matrix) that breaks the $SU(3)$ [2]. This reasoning led to the understanding of baryon level spacings–the Gell-Mann-Okubo mass formula, and many other successes in understanding strong and electromagnetic interactions [3]. The Gell-Mann-Okubo mass formula shows that possible modifications of baryonic wave functions, caused by the difference between the strange and light quark masses, do not modify the spectrum. Our purpose here is apply the old theoretical insights to the two-baryon system. In particular we shall argue for the likely existence of a $^1S_0$ loosely bound state of two cascade particles– the di-cascade. Then we shall discuss the newly feasible reactions that allow this system to be detected. Finding such a strangeness -4, baryon 2 system would be the discovery of a new dibaryon particle. Its existence would verify the flavor symmetry of the u, d and massive s quark interactions for systems of two baryons in the same irreducible representation of $SU(3)_F$. This would provide insight into how QCD works. The ability to understand strange nuclear matter would be increased and impetus would be given to lattice QCD studies of two baryon interactions[4].

The first step is to realize that $SU(3)$ flavor symmetry predicts the equality of the $^1S_0$ strong nucleon-nucleon NN interaction with the $^1S_0$ $\Xi\Xi$ strong interaction because the NN and $\Xi\Xi$ systems are each in the $\{27\}$ dimensional irreducible representation of $SU(3)$ [5]. This equality, the known existence of a quasibound state in the $^1S_0$ NN channel, and the increase of the reduced mass in the $\Xi\Xi$ channel, makes it likely that the $^1S_0$ $\Xi\Xi$ state is bound.

We next discuss the equality of the $^1S_0$ and $\Xi\Xi$ $^1S_0$ strong interactions. It is convenient to use the formalism of Savage & Wise[6]. The baryon fields are introduced as a 3×3 octet matrix

$$B = \begin{bmatrix} \Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} & \Sigma^+ & p \\ \Sigma^- & -\Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{bmatrix} , \qquad (1)$$

that transforms under chiral $SU(3)_L \times SU(3)_R$ as $B \to UBU^\dagger$.

The properties of low energy baryon-baryon interactions can be described by terms in $\mathcal{L}$ with four baryon fields (and no derivatives). They are given by

$$\begin{aligned}\mathcal{L}^{(2)} = & - c_1 Tr(B_i^\dagger B_i B_j^\dagger B_j) - c_2 Tr(B_i^\dagger B_j B_j^\dagger B_i) \\ & - c_3 Tr(B_i^\dagger B_j^\dagger B_i B_j) - c_4 Tr(B_i^\dagger B_j^\dagger B_j B_i) \\ & - c_5 Tr(B_i^\dagger B_i) Tr(B_j^\dagger B_j) - c_6 Tr(B_i^\dagger B_j) Tr(B_j^\dagger B_i) \quad ,\end{aligned} \tag{2}$$

where the indices i, j represent the spin of the two-component baryon fields, and repeated indices are summed over. In writing Eq. (2) we have ignored the explicit effects of the exchange of the pseudo-Goldstone bosons, terms of higher order in the chiral expansion and $SU(3)_F$ breaking terms. We shall return to all of those below.

The key feature of our present interest is that the nucleon and cascade doublets occupy analogous positions in the baryon matrix Eq. (1). Therefore the interaction Eq. (2) is invariant under the transformation $NN \leftrightarrow \Xi\Xi$. Evaluation of Eq. (2), using the properties of the Majorana exchange operator $B_j B_i = \frac{1}{2}(1 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) B_i B_j$, and keeping only the cascade and nucleon states leads to the result:

$$\begin{aligned}\mathcal{L}^{(2)}_{N,\Xi} = & - \left(c_1 + c_5 + \frac{1}{2}(c_2 + c_6)\right)\left((\Xi^\dagger\Xi)^2 + (N^\dagger N)^2)\right) \\ & - (c_2 + c_6)\frac{1}{2}\left(\Xi^\dagger \boldsymbol{\sigma}\Xi \cdot \Xi^\dagger \boldsymbol{\sigma}\Xi) + N^\dagger \boldsymbol{\sigma} N \cdot N^\dagger \boldsymbol{\sigma} N)\right) \\ & - 2(c_3 + \frac{1}{2}c_4)\Xi^\dagger N^\dagger N \Xi + c_4 \Xi^\dagger \boldsymbol{\sigma} N \cdot N^\dagger \boldsymbol{\sigma}\Xi.\end{aligned} \tag{3}$$

Equation (3) makes clear the prediction of the equality of the $\Xi\Xi$ and NN interactions, already present in Eq. (2). The interactions of Eq. (3) refer to both the ${}^1S_0$ and ${}^3S_1$ channels. However the one pion exchange interaction has a big influence in the triplet channel, and a much smaller influence in the singlet channel. Indeed, KSW[7] counting (in which the pion exchange interaction is treated as a perturbation) may be applied to the ${}^1S_0$ but not the ${}^3S_1$ channel [8]. Furthermore the NN and $\Xi\Xi$ ${}^3S_1$ states do not belong to the same irreducible representation of $SU(3)$ [5]. Therefore we shall consider only the ${}^1S_0$ channel, and neglect the explicit effects of one boson exchange as well as the possibly important effects of flavor symmetry breaking of interactions in the schematic calculations we present here. Both of these effects are taken into account in realistic calculations [5] that have obtained the same conclusions that we shall reach below.

The Lagrangian Eq. (3) must be extended by including the kinetic energy term $\mathcal{L}_{KE} = -Tr B_i^\dagger (\nabla^2/2M_B) B_i$. In addition there are four-baryon terms that involve derivative operators. We shall not include such terms (the d_i terms) directly. Our purpose here is to obtain simple interactions and then show that the use of different reduced masses in the Schroedinger equation can lead to bound states. Therefore we study the low energy regime in which the interaction is well described by the scattering length a and effective range r_e so that the phase shift $\delta(k)$ can be expressed as

$$k \cot\delta = -\frac{1}{a} + \frac{1}{2} r_e k^2. \tag{4}$$

In this regime including the effects of the d_i terms is indistinguishable from using a nucleon-nucleon simple potential that is defined by a depth and a range. Here we shall consider three potentials: square well of depth V_0 and range R, a non-local separable potential of the form $V(r, r') = -\frac{\lambda}{2\mu}u(r)u(r')$, where $u(r) = \frac{e^{\frac{-r}{b}}}{r}$, μ is the NN effective mass, and a delta-shell potential $V(r) = -\lambda\delta(r - R)$. The latter two are taken from the text by Gottfried[9]. One may choose the depth and range parameter to reproduce the scattering length and range of the ${}^1S_0$ system. We use either $a = -18$ fm (from the average of the nn and pp systems, set I) or $a = -24$ fm from the np system (set II). In either case we take $r_e = 2.8$ fm. The large magnitude and negative sign of the scattering length indicates the presence of a quasibound state: a slightly stronger interaction between nucleons would have caused a bound state to appear and the scattering length to be positive. The Schroedinger equation in the ${}^1S_0$ channel can be expressed (for a local potential) as

$$-\frac{d^2u}{dr^2} + 2\mu V u = k^2 u, \tag{5}$$

where $u(r)/r$ is the wave function. If the $\Xi\Xi$ interaction is the same as the nucleon-nucleon interaction V, then using the appropriate $\Xi\Xi$ reduced mass corresponds to a forty percent increase in the strength of the interaction. Alternately, the square of the effective momentum inside the well $k^2 - 2\mu V$ would be increased.

Obtaining analytic expressions for the scattering length and effective range is a straightforward matter for each of the potentials we employ. The results for the square well sq are:

$$a_{sq} = R\frac{x\cot x - 1}{x\cot x},\ r_e^{sq} = \frac{3x + (3 - 6x^2)\cot x + x(-3 + 2x^2)\cot^2 x}{3x(-1 + x\cot x)^2}, x \equiv R\sqrt{2\mu V_0} \tag{6}$$

To achieve a negative scattering length of very large magnitude the value of x must be slightly less than $\frac{\pi}{2}$. If the value of x were to be slightly greater than $\frac{\pi}{2}$, then a would be positive and a bound state would exist. We find that $x = 1.48, R = 2.64$ fm to reproduce set I, or $x = 1.50, R = 2.68$ fm, to reproduce set II. If $SU(3)$ flavor symmetry holds for the interaction V the values of of $x_{\Xi\Xi}$ for the cascade system would be either 1.76 (set I) or 1.78 (set II). These correspond to scattering lengths of 10.6 fm, and 9.81 fm, and binding energies of 7.48 MeV and 6.83 MeV.

The analysis of the separable potential sep case proceeds in a similar manner with similar results. The results for the phase shift are in ref. [9]:

$$a_{sep} = \frac{2\xi b}{\xi - 1},\ r_e^{sep} = \frac{b(2 + \xi)}{\xi},\ \xi = 2\pi\lambda b^3. \tag{7}$$

To achieve a negative scattering length of very large magnitude the value of ξ must be slightly smaller than one. If the value of ξ were to be slightly greater than one, then a would be positive and a bound state would exist. We find that $\xi = 0.911, b = 0.88$ fm to reproduce set I, or $\xi = 0.931, b = 0.89$ fm, to reproduce set II. If $SU(3)$ flavor symmetry holds for the interaction V the values of of $\xi_{\Xi\Xi}$ for the cascade system would be either 1.28 (set I) or 1.30

(set II). These correspond to scattering lengths of 8.0 fm, and 7.6 fm. The binding energy $B = \frac{\alpha^2}{2\mu_{\Xi\Xi}}$ is obtained by solving the equation

$$1 = \xi_{\Xi\Xi} \frac{1 - 2y + y^2}{(y^2 - 1)^2}, \; y \equiv \alpha b. \tag{8}$$

We find $\alpha b = 0.131$ and 0.140 that correspond to binding energies of 0.66 MeV and 0.73 MeV.

The analysis of the delta-shell potential *dsh* case is also similar. The phase shifts are presented in Ref. [9], with

$$a_{dsh} = R\frac{\gamma}{\gamma - 1}, \; r_e^{dsh} = \frac{\frac{2}{3}R(1+\gamma)}{\gamma}, \; \gamma \equiv \lambda R. \tag{9}$$

To achieve a negative scattering length of very large magnitude the value of γ must be slightly smaller than one, with a value slightly greater than one corresponding to the existence of a bound state. We find that $\gamma = 0.930, R = 2.02$ fm to reproduce set I, or $\gamma = 0.946, R = 2.04$ fm, to reproduce set II. If $SU(3)$ flavor symmetry holds for the interaction V the values of of $\gamma_{\Xi\Xi}$ for the cascade system would be either 1.30 (set I) or 1.32 (set II). These correspond to scattering lengths of 8.75 fm, and 8.34 fm. The binding energy $B = \frac{\alpha^2}{2\mu_{\Xi\Xi}}$ is obtained by solving the equation

$$\gamma_{\Xi\Xi} = \alpha R(1 + \coth \alpha R). \tag{10}$$

We find $\alpha R = 0.275$ and 0.291 that give binding energies of 0.549 and 0.606 MeV.

For each of the models considered, the use of a given potential combined with changing the reduced mass from that of NN to that of $\Xi\Xi$ leads to the prediction of a positive scattering length and the existence of a bound state. The scattering lengths range from about eight to eleven fm, while the binding energies have a much wider range from about 0.55 to 7.5 MeV. These calculations use very simple potentials, but the argument is clear. Increasing the magnitude of a potential that just misses having a bound state by forty percent should lead to the existence of a bound state.

It is necessary to discuss the effects of including the pseudo-Goldstone bosons that appear in chiral perturbation theory as well as $SU(3)$ breaking terms that enter in the interactions. These effects are included in Ref. [5] which obtained soft-core baryon baryon potentials for the complete baryon octet using the formalism of Ref. [10]. The potentials are parameterized in terms of one-boson exchanges. Boson-nucleon form factors are included to handle the short-distance part of the interaction. The form factors depend on the $SU(3)_F$ assignment of the mesons. The $^3P_0$ mechanism is used to generate the flavor-symmetry breaking of the coupling constants. Six different models of the hyperon-nucleon interaction that describe the data equally well are constructed in Ref. [10]. All of the parameters of each model are fixed in Ref. [10] so that each defines a baryon-baryon model that models all possible baryon-baryon interactions [5]. Each of the six potentials predicts the existence of a $\Xi\Xi$ bound state in the $^1S_0$ channel. The binding energies range from 0.1 to 15.8 MeV, a variation that is similar to that obtained using simple potentials.

Note also that the existence of $\Lambda\Lambda$ hypernuclei, taken along with $SU(3)_F$ symmetry, implies that the hyperon-hyperon interaction is strongly attractive [11]. One may construct one boson exchange potentials that reproduce the strong attraction in the $\Xi\Xi$ channel, providing another model[11].

We have seen that a $\Xi\Xi$ $^1S_0$ bound state (di-cascade) occurs in at least six realistic and three simple (a total of nine) different potential models. Its existence is therefore more than plausible, so we next comment briefly about properties and methods of detection. These loosely-bound di-cascade states would decay by the weak interaction to $NN4\pi$ final states. The lifetime would be roughly that of a free Ξ, about 2×10^{-10} s. For a discussion of other weak decay modes see Ref. [12]. Furthermore, the small binding energies tell us that the di-cascade consists of two well-separated baryons and therefore is fragile and easily absorbed if produced in a reaction that surrounds it with nucleons.

We concentrate on the $\Xi^0\Xi^-$ or $\Xi^0\Xi^0$ systems because the repulsive effects of the Coulomb interaction could cause a state, weakly bound under the strong interaction, to be unbound. Reactions involving two baryons seem best suited for the discovery of the $^1S_0$ bound state. Therefore it seems promising to make the search at Jefferson Laboratory using the missing-mass technique in the reactions

$$\gamma + D \rightarrow (\Xi^0\Xi^-)_{^1S_0} + K^+ + K^+ + K^0K^0 \tag{11}$$
$$\gamma + D \rightarrow (\Xi^0\Xi^0)_{^1S_0} + K^+ + K^0 + K^0K^0. \tag{12}$$

The photon threshold energy is about 4.8 GeV. This reaction presents the difficulty of measuring four kaons, but there should be a clear signature as a sharp peak in the missing-mass spectrum. The future availability of high intensity K^- beams at J-PARC makes it interesting to consider the reactions

$$K^- + D \rightarrow (\Xi^0\Xi^-)_{^1S_0} + +K^+ + K^0K^0 \tag{13}$$
$$K^- + D \rightarrow (\Xi^0\Xi^0)_{^1S_0} + +K^0 + K^0K^0. \tag{14}$$

The kaon threshold energy is 3.8 GeV, and one would need to detect only three kaons in the final state.

Both the photon and kaon induced searches would require much experimental effort to find the $(\Xi\Xi)_{^1S_0}$ di-cascade bound state. However, the recent observation of the Ξ^- in the γp reaction at Jefferson Laboratory and the expected availability of high-intenstiy kaon beams at J-PARC make it evident that the necessary experimental tools exist or can be obtained. This bound state is predicted to exist in nine different models, so that a careful search is likely to be successful. However, models can not provide a definitive proof that the state of interest exists. Therefore we call for the development of experiments capable of determining whether or not the $(\Xi\Xi)_{^1S_0}$ di-cascade bound state exists.

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